

Exercise 27

For the following exercises, use the Intermediate Value Theorem to confirm that the given polynomial has at least one zero within the given interval.

$$f(x) = -x^4 + 4, \text{ between } x = 1 \text{ and } x = 3.$$

Solution

Plug $x = 1$ and $x = 3$ into the function.

$$f(1) = -(1)^4 + 4 = 3$$

$$f(3) = -(3)^4 + 4 = -77$$

Since $f(x)$ is a polynomial function (a smooth and continuous function), $f(x)$ has to take on every value between -77 and 3 for $1 < x < 3$ by the Intermediate Value Theorem. Therefore, $f(x)$ has a zero between $x = 1$ and $x = 3$.